

## MAGNETIC MOMENT IN THE MAGNETIC FIELD

**This is the outline of the material required. Further information is provided in the proposed literature.**

A conductor loop carrying a current  $J$  forms a magnetic field with induction  $\vec{B}$ . Direction of vector  $\vec{B}$  depends on the direction of the current flow in the circuit.

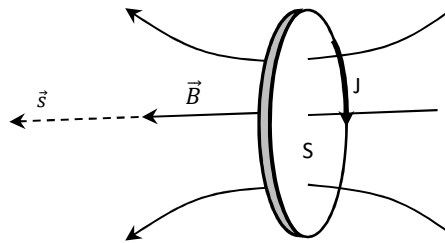


Fig.1. Shape and direction of magnetic field lines around a conductor loop carrying a current  $J$ .

Magnetic moment  $\vec{\mu}$  is the vector quantity commonly used to characterize the magnetic field formed by the closed electrical circuit. It is defined as the product of current and the area of the closed circuit, represented by vector  $\vec{s}$  normal to the surface:

$$\vec{\mu} = I\vec{s} \quad (1)$$

The magnetic moment of a single coil of a circular current loop with the radius  $R$  can be calculated:

$$\mu = I \cdot \pi R^2 = \frac{\pi}{4} Id^2, \quad (2)$$

Where  $d$  is the diameter of the circle.

In case of multiple turns, their number ( $n$ ) has to be included in formula (2) which takes the following form:

$$\mu = n \cdot I \cdot \pi R^2 = n \cdot \frac{\pi}{4} Id^2 \quad (3)$$

When the current loop is placed in the external uniform magnetic field of induction  $\vec{B}$ , it will experience a torque  $\vec{M}$  defined below:

$$\vec{M} = \vec{\mu} \times \vec{B}, \quad (4)$$

where „ $\times$ ” denotes vector product. Based on the properties of vector product, the value of force moment (torque) can be calculated with the following formula:

$$M = \mu \cdot B \cdot \sin\alpha, \quad (5)$$

where  $\alpha$  is the angle between vectors  $\vec{\mu}$  and  $\vec{B}$ .

Formulas (3) and (5) can be combined to allow for the calculation of the value of the force moment as a function of the current  $J$  in the closed circuit placed in the uniform magnetic field of induction  $B$ :

$$M(I) = \frac{n\pi B a^2}{4} I \quad \text{for } \alpha = 90^\circ \rightarrow \sin\alpha = 1 \quad (6)$$

The uniform magnetic field in the experiment is produced by Helmholtz coils (Fig.2).

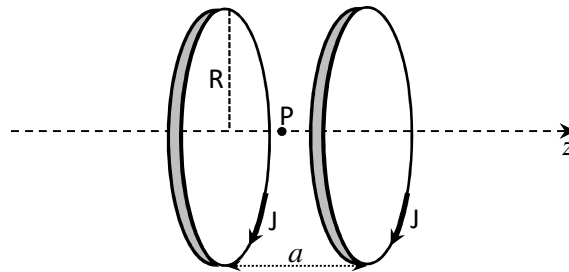


Fig.2. Helmholtz coils

The value of magnetic field induction on the axis of Helmholtz coils can be calculated on the basis of Biot's-Savart's law. For point P lying at a distance of  $a/2$  from the planes of the two coils (in the middle) the formula takes the form:

$$B_z = \frac{\mu_0 n I R^2}{2} \left\{ \left[ R^2 + \left( z - \frac{a}{2} \right)^2 \right]^{-\frac{3}{2}} + \left[ R^2 + \left( z + \frac{a}{2} \right)^2 \right]^{-\frac{3}{2}} \right\} \quad (7)$$

In the equation above  $\mu_0$  is the magnetic permeability of vacuum,  $n$ - the number of turns in a coil,  $I$  - current,  $R$  - radius of coils,  $a$  - distance between the coils. It is convenient to assume that the coordinate  $z$  for point P  $z = 0$ .

The results of calculations show that the magnetic field between Helmholtz coils is most uniform, when  $a = R$  (Fig. 3.).

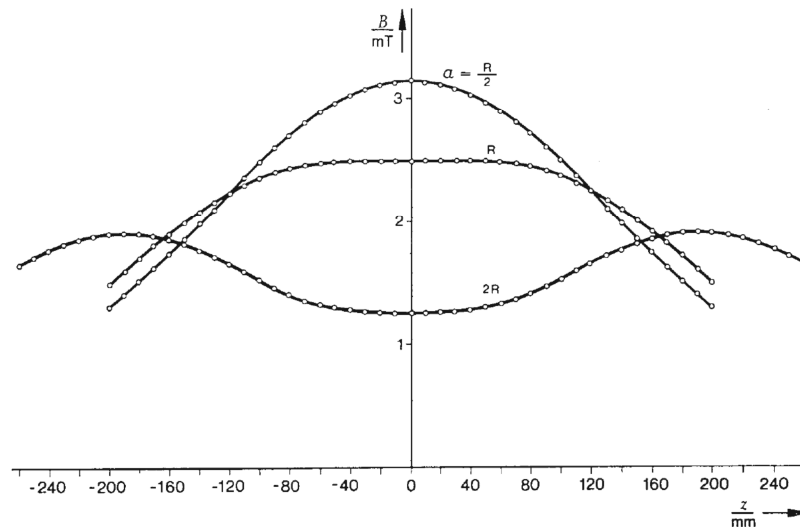


Fig.3. Plot of the induction of the magnetic field  $B$  on the axis of Helmholtz coils as a function of distance from the middle between the coil's planes for three different values of  $a$ .

For  $a = R$  and  $z = 0$  formula (7) can take the form:

$$B_z = 0,7155\mu_0 \frac{nI}{R} \quad (8)$$

### Required theoretical knowledge

1. Uniform magnetic field, induction of a magnetic field. Units of measure of the strength and induction of a magnetic field. Magnetic force acting on an electric charge moving in a uniform magnetic field. Magnetic force acting on a wire placed in a uniform magnetic field.
2. Pair of forces acting on a current-carrying rectangular frame placed in a uniform magnetic field.
3. Definition of a torque (moment of force) and of a magnetic moment (units of measure).
4. Calculation of a value of a magnetic moment for a circuit with a circular shape.
5. Calculation of a torque exerted by the magnetic force on a current-carrying circuit placed in a uniform magnetic field.
6. Structure of the setup (Helmholtz coils) used to measure forces acting on a current-carrying circuit placed in a uniform magnetic field.
7. Absorption of electromagnetic waves - transitions between energy levels of paramagnetic nuclei in a constant magnetic field with the induction of  $B$ . The phenomenon of a nuclear magnetic resonance (NMR). Resonance frequency.
8. The NMR spectroscopy and its application in chemistry, biology and medicine. Functional NMR and its application in a medical diagnostics.

### Literature

1. D. Halliday, R. Resnick, J. Walker: „Fundamentals of physics”, Wiley 2015.